

# Correlation Breakdowns, Spread Positions, and CCP Margin Models

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## Abstract

The default of a participant at Nasdaq Clearing in 2018 and the recent COVID-19 events brought to the attention of risk managers at CCPs the importance of appropriately measuring correlation breakdowns. The sizable price dislocations registered on these occasions suggested that traditional risk models may not be fully equipped to capture the breakdowns. Because correlations are directly impacted by the statistical properties of each variable, any model that lacks the capacity to deal with non-stationarity may inappropriately represent correlation or its alterations. Using a GARCH-DCC approach to accommodate such properties, the objective of the paper is to study the correlation behaviour during adverse market conditions, and the potential subsequent impact to CCP margins. A study case for energy commodities is proposed, with the specific focus on spread positions for the electricity market. The analysis suggests that the correlation breakdowns are more frequent than traditionally expected. When different types of shocks are considered (i.e. September 2018 and March-May 2020), it becomes evident that while the magnitude of the breakdown may differ, its cycle presents a number of similarities. While elevated margin due to correlation breakdown may reduce breaching amount and improve margin coverage rate, this paper also recognizes the potentially increased margin procyclicality, and highlights the challenge of balancing margin responsiveness and stability during correlation breakdown for spread positions, and calls for further study in this area.

**Keywords:** Correlation, Multivariate t-Copula, VaR, Expected Shortfall, Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC), Historical Simulation (HS), Filtered Historical Simulation (FHS), Spread Positions, Nasdaq, COVID, Volatility Forecast, Correlation Forecast, Covariance Forecast

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## 1. Introduction

The default at Nasdaq in September 2018 and the recent COVID-19 related volatility between March and May 2020 brought to the attention of risk managers at central counterparties (CCPs) the importance of appropriately measuring tail risks for correlation breakdowns, in particular for strategy-based portfolios. An extreme variation on September 10 2018 led the price spread between German and Nordic Power futures to reach €5.56/Mwh, compared to the previous high of €1.85/Mwh. Similar extreme moves related to the abrupt changes in the correlation structure of price returns were also observed during the initial shocks of the COVID-19 pandemic in a number of markets. For instance, the spread between the first and second expiry in WTI contracts jumped close to 900% on April 20<sup>th</sup> 2020 when compared to previous day.<sup>1</sup>

Instances of correlation breakdowns are not new to financial markets. In fact, after almost all major crises of the past 30 years studies have been performed to better understand the reasons leading to and the impacts of a rupture in the dependency between different prices and returns. However, it is remarkable that as markets develop and alter the way demand and supply are established, so does the manner in which correlations behave during times of adverse conditions. The earlier studies have shown that correlation breakdowns were typically associated to the phenomenon where the correlation for different assets would suddenly move close to the extremes. The recent examples of 2018 and 2020 have demonstrated that prices may not all go in the same direction when shocks hit, and any move away from an “expected” dependency between different prices can also have distressing effects.

In the CCP risk management world the importance of understanding such dislocations is no different. As presented in Vicente et al. (2015), the vast majority of the risk methodologies currently employed by CCPs trace their origins to a common forefather: the value-at-risk approach, VaR. This single risk figure concept strongly influenced the development of traditional CCP risk management models, as it could be easily translated into a margin requirement. However, as discussed by the authors, the standard implementations of the VaR approach may face some challenges to fully reflect the reality of multi-asset risk management in CCPs. In particular, when historical or parametric VaR are considered, the ability to deal with correlation breakdowns may be limited.

The objective of this paper is to study the correlation behaviour during adverse market conditions. In order to address the non-stationary statistical properties of price returns, the GARCH-DCC model is used. Boyer et al. (1999) have demonstrated that changes in correlation may simply be a reflection of alterations in the representation of the volatility of variables under analysis. As such, any model that lacks the capacity to deal with non-stationarity may inappropriately represent correlation or its alterations. As alluded previously, in the CCP margin model area, multivariate fat-tail based VaR model is one popular parametric method. However different configurations of such models can significantly influence the way correlation breakdowns are

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<sup>1</sup> The main driver of this change was the price of the first expiry of the WTI contract which decreased from close to \$18/barrel on April 17<sup>th</sup> to approximately \$-38/barrel on April 20<sup>th</sup>.

considered. Li and Cheruvelil (2019) discuss some of these features highlighting how the introduction of correlation ceilings could limit some of the impacts of the breakdowns.<sup>2</sup>

In order to analyse the correlation behaviour a study case for energy commodities is proposed, with a specific focus on the electricity market. Because these commodities cannot be stored in a flexible and cost effective manner, preventing rapid responses to sudden changes in supply and demand, they are good candidates in the study of shocks to correlation. And the events of September 2018 and March/May 2020 allow us to contrast such behaviour in response to two distinct types of shocks. While the former is mainly endogenous and to a large extent related to climate conditions affecting the electricity market only, the latter can be viewed as an exogenous macroeconomic shock spreading across several asset classes.

Using spread positions on futures contracts as the mechanism to capture the dependency between different assets, the paper is able to confirm some of the well-known features of the energy markets, such as excess kurtosis (i.e. fat tails), jumpiness and non-stationarity (see Carmona and Durrelman (2003) for a discussion of these features). The implementation of an enhanced multivariate GARCH model enables the identification of the correlation breakdowns, suggesting that these dislocations are more frequent than traditionally expected. When the different types of shocks are considered, it becomes evident that while the magnitude of the breakdown may differ, its cycle presents a number of similarities. The above enhanced methodological approach also suggests that traditional risk models may face challenges for capturing the dynamics of sizable spread position movements.

In addition to this introduction, the paper is organized as follows. Section 2 presents the literature review. Section 3 introduces the study case and a potential enhancement to the standard VaR approach that could filter more appropriately the statistical properties of spread positions. Section 4 displays the results for the empirical implementation of the enhanced VaR. Section 5 concludes the paper.

## 2. Literature review

### 2.1. Correlation and its intrinsic features

Correlation is a crucial aspect of the mechanics of many derivatives and assets in the financial markets. However, it is not a novel fact that correlation presents limitations as a metric of dependence between two distinct statistical variables. For instance, correlation is a linear measure of dependency, and any relationship structure that is non-linear might not appropriately be captured. This is the case of the correlation of a variable  $X$ , normally distributed, and  $X^2$ , Chi-

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<sup>2</sup> This research is not an attempt to advocate for or promote a specific margin model or methodology (i.e. GARCH-DCC Framework) for use by any CCP; such research would involve a different analysis, inclusive of other considerations such as efficiency and procyclicality effects. However, this research could potentially help practitioners gain insights into the nature of correlation behavior for risk management purposes

square. Even though one is a function of the other, correlation between them is zero. Another relevant feature is that correlation does not necessarily vary between 1 and -1. In fact, the range of the correlation values depends on the statistical distribution of the variables where the dependency is being measured. The closer these distributions are to what is called the elliptical family (e.g. Normal, t-Student, etc.) the more appropriate the metric becomes.

Although the Frechet-Hoeffding bounds were established years ago, the minimum and maximum correlation values are frequently not part of studies of dependency.<sup>3</sup> The increasing interest in Copulas certainly enhanced the awareness of these properties (see Nelsen 1990). Nonetheless, one of the issues of not taking into account the correlation boundaries is that results can be suboptimal and conclusions inappropriately derived. For instance, the larger the volatility of the individual variables, the smaller the chances of the correlation reaching a value close to the extremes. In this case, a correlation value not close to 1 and -1 does not necessarily mean that the dependency is irrelevant.

The above features suggest that care is needed when analysing correlations and its breakdowns. Eydeland and Wolyniec (2003) highlighted that correlations may have an unexpected behaviour due to four general reasons: i) change in conditional correlation, where the correlation coefficient might be time-dependent or stochastic; ii) inexistence of an unconditional correlation; iii) non-linear structure of dependency; and iv) estimation noise. Moreover, when confidence intervals given by the Fisher Transformation are considered, any appropriate estimation of correlations would require typically over 100 points before it starts to stabilize, although this number may increase for lower correlation levels.<sup>4</sup>

## 2.2. Correlation breakdowns<sup>5</sup>

Correlation breakdowns represent a problem for investors, risk managers, and the financial markets more broadly due to the fact that any risk protection endowed by portfolio diversification is lost. In the early literature, the term correlation breakdown was typically used to refer to the phenomenon where the correlation between the returns of different assets would suddenly move close to the extremes (see Bertero and Mayer (1990), King and Wadhvani (1990), and Boyer (1997)). Since then, and as the recent examples of Mr. Aas in 2018 and COVID-19 in 2020 show, the case where correlations move away from an “expected” behaviour can also have distressing impacts. And this can in fact mean that correlation simply disappears.

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<sup>3</sup> Fréchet, M. (1951). Sur les tableaux de corrélation dont les marges sont données. *Annales de l'Université de Lyon. Section A: Sciences mathématiques et astronomie* 9: 53–77. Hoeffding, W. (1940). Masstabinvariante Korrelationstheorie. *Schriften des Mathematischen Instituts und des Instituts für Angewandte Mathematik de Universität Berlin*, 5, 179-233. [Reprinted as: Scale-invariant correlation theory In: Fisher, N. I. and Sen, P. K., editors, (1994). *The Collected Works of Wassily Hoeffding*, 57-107. Springer, New York.

<sup>4</sup> Fisher, R. A. (1915). Frequency distribution of the values of the correlation coefficient in samples of an indefinitely large population. *Biometrika*. 10 (4): 507–521.

<sup>5</sup> In the literature different terminologies are used to refer to alterations in the dependency between distinct assets following the occurrence of a shock. In addition to correlation breakdown, asset contagion, run to unity, amongst others can be found.

The broadening of the concept of correlation breakdowns seems to reflect the empirical evidence that during periods of adverse market conditions correlations can have heterogeneous behaviour. King and Wadhvani (1990) analysis showed that the correlation of stock returns increased substantially at the time of the 1987 crash. More recently evidence is presented in the Bank of International Settlements 2017 Quarterly Review (see BIS (2017)). The report considers the institution's long-term monitoring of correlations and, conversely to the earlier studies, it is observed that returns became less dependent as policy uncertainty jumped to the foreground during 2016-2017.

Theoretically, different approaches have been pursued in order to explain the origins of correlation breakdowns. In the seminal work of Loretan and English (2000), the authors suggest that periods with high correlations tend also to be periods with higher than average volatility. The authors emphasise that correlation breakdowns reflect the time-varying volatility pattern of financial markets, rather than changes in the relationship of asset returns. When there exists a linear dependency between variables, the conditional correlation becomes a function of the volatility of the underlying. If the volatility in a particular time increases above its unconditional value, so will the correlation. Aiming at circumventing the effects of non-stationarity in the assessment of correlation different approaches have been proposed. Chang and Cheng (2016), for instance, consider a vector auto-regression (VAR) approach. Under such framework, the authors suggest that linkages between assets tend to increase following market shocks.

Another theoretical stream of work that tries to explain correlation breakdowns is based on the understanding of the behaviour of investors. Falbo and Grassi (2015), for example, propose that correlation breakdowns are related to the way rational agents correct their estimation of the correlation due to excess demand in periods of high volatility. Specifically, the model developed by the authors suggest that the herding behaviour of speculators and the procyclical attempt of rational investors to protect against the dislocations are the root-causes of the correlation breakdowns. In addition to these reasons, Dornbusch et al. (2000) suggests that restrictions to institutional investors' portfolios may also intensify the breakdowns, as these investors might have to recalibrate portfolios to comply with law or contractual clauses.

### **2.3. Spread positions**

The previous sections highlighted that correlations in financial markets may oscillate over time, making it challenging to model. The analysis of spread positions can be a key mechanism to better understand the behaviour of the correlation across time. In finance spreads broadly relate to one, or a set of contracts, which final value is based on the difference between prices of two or more distinct risk factors (in some cases, spreads are based on a finite linear combination of three or more of these risk factors). Typically this type of contract, or portfolio of contracts, is used to mitigate adverse movements in several risk factors at the same time. Defined as such, the alterations to the value of the spread can reveal important information about how correlation is expected to evolve.

In order to model the spread time series distinct approaches can be found in the literature. Jump-diffusion models gained popularity in the 1980s due to their ability to replicate, at least qualitatively, the spiky nature of spread prices. Initially applied to a single asset time series, they soon expanded to other types of data such as spreads (see Bates (1988) for a review of the literature). However, the inefficient and unstable estimates of the jump properties made the attempts not long-lasting. The instability was stemming from a combination of problem misspecification and the lack of extreme events in the time series. Moreover, when estimating the model, the parameters of jump distribution can be biased by the presence of stochastic volatility.

Another approach that was developed is based on the option pricing literature, where different proposed dynamics for the spread time series can be observed. One first modelling possibility is to assume that the spread behaves as a Normal distribution and the instantaneous change as an arithmetic Brownian motion. The supporting argumentation for this approach is based on empirical evidence, as the plotting of spread histograms tends to resemble Normal distributions in a few cases. This framework leads to the well-known Bachelier option pricer. Importantly, the assumption of an arithmetic Brownian motion for the spread is un-reconcilable with the standard view that prices behave as log-normal distributions, with their dynamics given by geometric Brownian motions (see Goldenberg (1991)).

Due to the above inconsistency, an alternative approach is to model each leg of the spread and define a dependency structure between them. Each underlying of the spread is modelled as a geometric Brownian motion and a correlation between two individual Wiener processes is used to represent the relationship between them. There is little argumentation that the use of geometric Brownian motion is an appealing approach when modelling energy commodity contracts and spread positions more specifically. Not only is the model commonly used in finance, but it also allows for a straightforward derivation of prices for spread options. One of the issues of using the option-derived geometric Brownian motion to imply the behavior of the underlying contracts regards the past versus future nature of the volatility. The key challenge of the geometric Brownian motion, however, is its limited capacity to replicate the statistical properties of contracts that exhibit excess kurtosis, skewness, and auto-regressive pattern of the variance.

For the purposes of this paper, the important aspect of such challenges is the fact that approaches with higher dimensionality such as the generalized autoregressive conditional heteroskedasticity (GARCH) model became more popular (see Engle (1982) and Bollerslev (1986)). For instance, using the GARCH specification with t-distributed (fat-tailed) innovations can improve the in-sample fit, helping to remove unconditional skewness and excess kurtosis from the data only. It is because of these and other important characteristics of the GARCH model to accommodate multivariate distributions that it is selected as a contender in the modelling of the spread contracts. In particular for the energy markets, as introduced in the next section, a number of papers have emphasized the positive features of the GARCH approach (see Zanottia et al. (2010), and Du and Lai (2017)).

### 3. Theory and case study

The energy markets have seen rapid changes in the last decades since the so-called deregulation phase. In particular, following the opening for competition on fuel markets in 1980s, in electricity in 1990s, and weather/emissions in late 1990s, the whole energy complex was marked by an expansion on trading volumes, contract types, and market participants (see Eydeland and Wolyniec (2003)). Together with these developments, new price discovery mechanisms were established, changing substantially the dynamics of price formation and its behaviour through time. In a few words, demand and supply emerged to replace the previous target-approach to valuation of energy commodities. In this new world a number of features became prominent in defining the dynamics of energy prices, such as seasonality, mean reversion, excess kurtosis and jumpy behaviour. Any risk model not able to capture these statistical properties may fail to perform as expected. The study case focuses on this energy market, in particular spread positions, and uses the GARCH-DCC approach in order to model its statistical properties.

#### 3.1. Data

According to Carmona and Durrelman (2003) spreads are probably the most useful, prevalent, and important structure in the world of energy. Spreads are used to describe power plants, refineries, storage facilities, and transmission lines. In some cases these spreads are used as a way to quantify the cost of production of refined products from the complex of raw material used to produce them. Practically every aspect of energy production and delivery can be explained using spreads. In order to study the dynamics of spread positions, the historical series of a portfolio composed of two energy futures contracts was assessed. Specifically, the ICE Endex German Power Financial Base Futures (GER) and the ICE Endex Nordic Power Financial Base Futures (NBR) were considered in the analysis. These are financially settled futures contracts based on monthly futures, with their valuation dependent on the hourly prices of electricity arising from their controlled areas.

Data used in the assessment are daily prices of these two contracts, covering the 12 first expiries and the period from June 6, 2016 to June 1, 2020. The period includes the week of September 2018, when prices for the spread between GER and NBR spiked to levels not previously observed, and the COVID-19 period. The extreme variation on September 10 2018 led the price of the spread to reach €5.56/Mwh, compared to the previous high of €1.85/Mwh. The dislocation was due to a combination of excessive rainfall in the Nordic regions that sharply increased hydropower supply, and concurrent price surge of the European carbon allowance which drove the price of German power high. While the 2018 shock was endogenous and affected the electricity market only, the COVID period was a global and systemic market stress, exogenously spreading across several asset classes.

#### 3.2. Methodology - Enhanced multivariate VaR

Performing the appropriate statistical analysis in energy commodities entails careful consideration due to the reasons alluded previously. Adjusting for these properties in search of stationarity is no simple task and any risk model that fails to do so increases substantially the chances of delivering poor statistical estimates.

### 3.2.1. GARCH-CCC

Bollerslev (1990) proposes a multivariate GARCH model using time-varying conditional variances and covariance but with constant correlations. The conditional covariance matrix is given by:

$$\mathbf{H}_t = \mathbf{D}_t \bar{\mathbf{R}} \mathbf{D}_t \quad (1)$$

where  $\mathbf{D}_t$  is a  $n \times n$  stochastic diagonal matrix with elements  $\sigma_{i,t}$ , which follows a univariate GARCH process and  $\bar{\mathbf{R}}$  is  $n \times n$  time-invariant unconditional correlation matrix of the standardized error  $\boldsymbol{\epsilon}_t$ .

$$\mathbf{D}_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \dots, \sigma_{n,t}) \quad (2)$$

$$\boldsymbol{\epsilon}_t = \mathbf{D}_t^{-1} \boldsymbol{\epsilon}_t \quad (3)$$

$$\bar{\mathbf{Q}} = \text{Cov}(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t^T) = E[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t^T] \quad (4)$$

$$\bar{\mathbf{R}} = \text{diag}(\bar{\mathbf{Q}})^{-1/2} \bar{\mathbf{Q}} \text{diag}(\bar{\mathbf{Q}})^{-1/2} \quad (5)$$

where  $\sigma_{i,t}^2$  follows GARCH process as defined in (2).  $\boldsymbol{\epsilon}_t$  is the de-autocorrelated residual as defined in (1) above.

The estimation of GARCH-CCC is computationally attractive because the correlation matrix is constant. However, this correlation estimator may be too restrictive based on empirical evidences. The model needs to be generalized by assuming the correlation matrix varies with time.

### 3.2.2. GARCH-DCC and DCC-copula

The Dynamic Conditional Correlation (DCC) model was introduced by Engle and Sheppard (2001). The key design idea is that the dynamic covariance matrix  $\mathbf{H}_t$  can be decomposed into conditional standard deviations  $\mathbf{D}_t$  and a correlation matrix  $\mathbf{R}_t$ . Both  $\mathbf{D}_t$  and  $\mathbf{R}_t$  are time-varying.

The conditional correlation estimator under multivariate DCC representation is:

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2} \quad (6)$$

$$\mathbf{Q}_t = (1 - \alpha - \beta) \bar{\mathbf{Q}} + \alpha \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}^T + \beta \mathbf{Q}_{t-1} \quad (7)$$

where  $\mathbf{R}_t$  is the DCC correlation at time  $t$ .  $\bar{\mathbf{Q}}$  is the unconditional covariance matrix as defined in (4),  $\alpha$  represents dynamic term introduced by the interaction between the two innovations, and  $\beta$  represents persistence term. To ensure matrix  $\mathbf{R}_t$  is positive definite, the scalars  $\alpha$  and  $\beta$  must satisfy:

$$\alpha \geq 0, \quad \beta \geq 0, \quad \alpha + \beta < 1$$

The DCC-copula can be represented as:

$$\mathbf{F}(\epsilon_{1t}, \epsilon_{2t}, \dots, \epsilon_{nt}) = \mathbf{C}(\mathbf{F}_1(\epsilon_{1t}), \mathbf{F}_2(\epsilon_{2t}), \dots, \mathbf{F}_n(\epsilon_{nt}); \boldsymbol{\psi}_t) \quad (8)$$

where  $\boldsymbol{\psi}_t$  is the copula parameter including dependence structure parameter  $\mathbf{R}_t$  and the multivariate DoF or copula DoF  $\mathbf{v}_c$ .

To estimate the DCC-copula, there are generally two key separate steps by using Maximum Log-Likelihood. The log-likelihood is obtained from the following formula:

$$\begin{aligned} \ln(L(\theta)) = & \sum_{t=1}^T \left( \ln \left[ \Gamma \left( \frac{\mathbf{v}_c + \mathbf{n}}{2} \right) \right] - \ln \left[ \Gamma \left( \frac{\mathbf{v}_c}{2} \right) \right] - \frac{\mathbf{n}}{2} \ln[\pi(\mathbf{v}_c - 2)] \right. \\ & \left. - \frac{1}{2} \ln[|\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t|] - \frac{\mathbf{v}_c + \mathbf{n}}{2} \ln \left[ 1 + \frac{\boldsymbol{\epsilon}_t^T \mathbf{R}_t^{-1} \boldsymbol{\epsilon}_t}{\mathbf{v}_c - 2} \right] \right) \quad (9) \end{aligned}$$

The parameter set  $\theta$  is divided into two groups:

$$(\phi, \psi) = (\phi_1, \phi_2 \dots \phi_n, \psi)$$

where  $\phi_i = (\alpha_{1i}, \dots, \alpha_{ni}, \beta_{1i}, \dots, \beta_{ni})$  are the parameters of the univariate GARCH model for the risk factor,  $i = 1, \dots, n$  and  $\psi = (\alpha, \beta, \mathbf{v}_c)$

1. Estimate univariate GARCH to get  $\phi$ <sup>6</sup> for calculating  $\mathbf{D}_t$  and
2. Estimate  $\psi$  to simultaneously obtain time-varying dependence structure  $\mathbf{R}_t$  (i.e.  $\alpha$  and  $\beta$ ) and  $\mathbf{v}_c$  using standardized residual from the first step.

The advantage of DCC-copula is that the log-likelihood of the volatility and correlation can be maximized independently as long as consistence is ensured within these two steps (Engle (2002)).<sup>7</sup>

<sup>6</sup> The autocorrelation coefficient and individual DoF are also estimated in the first step.

<sup>7</sup> Technical limitations for DCC have been well discussed in the academic literatures, e.g. Caporin and McAleer (2013). One of the limitations is that the two step estimators in DCC may not be consistent because in equation (7) the matrix  $\mathbf{Q}_t$  is not the expectation of the standardized residuals cross-products. The various attempts for enhancements are beyond the scope of this paper.

## 4. Empirical assessment and modelling fitting

The historical price series of the first 12 monthly expiries for the German Power Financial Base Futures and the Nordic Power Financial Base Futures are displayed in Figure 1. The different statistical behaviors for each contract and maturity can be observed in the chart. It is also evidenced how the co-movement of GAB and NRB vary through time and for the distinct expiries. However, before exploring the correlation behavior, a brief summary assessment of the univariate and multivariate statistical properties of those series is presented below.

Figure 1 - Historical Price Series GAB and NRB



### 4.1. Univariate statistical analysis and model fitting

It has been well documented in the literature the specific statistical features of the energy markets such as excess kurtosis (i.e. fat tails), jumpiness, heteroskedasticity, amongst others. The study of the two return series corroborates the existence of such features. To illustrate the analysis the 12-month expiry (12M) is considered. In particular, it is observed that these series exhibit a significant fat tail with its Degree of Freedom (DoF) as between 2 and 3. Both GER and NRB price return time series show little skewness, but sizably kurtosis (3 is normal). The Q-Q test results also show significant deviation from normal distribution. See details in Tables A1 to A2 and Figures A1 to A2 in the Appendix.

To accommodate the non-stationarity of these series, several symmetric and asymmetric volatility and innovation models are tested for goodness of fit. The GJR-GARCH is selected for asymmetric volatility model, and Hansen's Skewed student-t is selected for asymmetric innovation model. For the time series up to September 7, 2018, which is the day before the September 10, 2018 Nasdaq event, symmetric GARCH(1,1) and symmetric student-t exhibit the best results comparing with other combinations. The combinations of either asymmetric GJR-GARCH or asymmetric Hansen's t produce slightly lower but comparable likelihood, however, the GJR term or the Skew term from Hansen's t is not statistically significant, which is consistent with the relatively small skewness exhibited in the statistical analysis of the empirical time series. Therefore, for the analysis herein, GARCH(1,1) and student-t are selected for univariate volatility and innovation modelling for both GAB and NRB. Both time series show little autocorrelations. See details in Tables A3 to A6 in the Appendix.

#### 4.2. Multivariate model calibration

For multivariate calibration, joint MLE method is used to calibrate the model as described in the Section 3.2. The multivariate dynamic copula model parameters,  $\alpha$ ,  $\beta$  and  $\nu_c$  are all statistically significant.<sup>8</sup> The multivariate DoFs also reflect reasonably well their respective univariate DoFs. All three parameters are stable by comparing before and after the Nasdaq event, as well as during the COVID period. The Samuelsson effect is not accounted for this time. The effect is not strong for long-term contract such as this one-year contract. Table 1 to 3 below illustrate these parameters for the 12-month expiry and specific days preceding sizeable shocks in the spread contracts.

Table 1: GARCH-DCCdynamic copula model parameters on September 7, 2018

Model parameters (DCC)	$\alpha$	$\beta$	$\nu_c$
Parameter value	0.0220972	0.8870032	2.70
Error	(0.01422)	(0.081214)	(0.07)

Table 2: GARCH-DCCdynamic copula model parameters on January 24, 2019

Model parameters (DCC)	$\alpha$	$\beta$	$\nu_c$
Parameter value	0.0154947	0.9006684	2.90
Error	(0.010254)	(0.0941639)	(0.07)

Table 3: GARCH-DCCdynamic copula model parameters on May 15, 2020

<sup>8</sup>  $\beta$  and  $\nu_c$  are statistically significant above 99%, the  $\alpha$  term is above 90% significance.

Model parameters (DCC)	$\alpha$	$\beta$	$\nu_c$
Parameter value	0.0188728	0.9014135	2.82
Error	(0.011134)	(0.0941639)	(0.06)

### 4.3. Correlation analysis

The appropriate calibration of the univariate and multivariate models enables one to deal with the non-stationarity of the return time series. As such, correlation can be studied without the undesirable effects of these on the dependency measurements. Under such approach, it is observed that the correlation between GAB and NRB fluctuates around the long-term historical average, as indicated by the unconditional correlations. This effect is prominent for the longer expiries, i.e. those equal or larger than 4 months, as presented in details in Figure A3 in the Appendix.

If correlation breakdowns are defined as dislocations beyond a 95% confidence interval around the long-term average, it is noticeable that several breaks happened throughout the analysed historical period. This evidence suggests that these dislocations are more frequent than traditionally expected, and the events of September 2018 and March/May 2020 are not exceptions. Moreover, different than commonly postulated, the breakdowns occur in both directions, and are not necessarily associated with the correlation values moving closer to the unit.

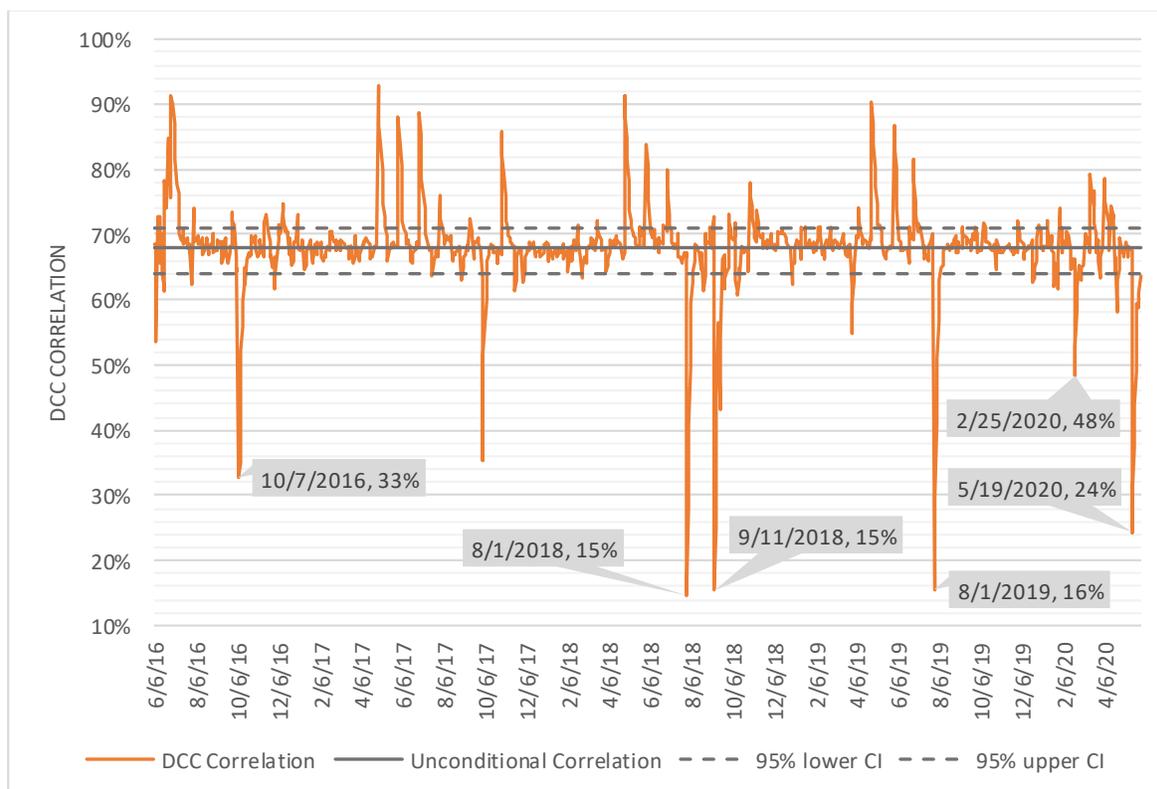
Another important observation is that correlation breakdowns are not homogeneous across different expiries. This is true for both shocks analysed and, even when the systemic stress of the COVID period is accounted for, no consistent pattern is recognized for contracts with different maturity dates. Similarly, the comparison between the effects of the shocks from September 2018 and March/May 2020 periods suggests that, while the magnitude of the breakdown may differ, its cycle presents a number of similarities. More specifically, after a dislocation is observed, in general the correlation tends to return quickly to its long-term average.

In order to illustrate the above aspects, the 12-month expiry is considered, as displayed in Figure 2. The 95% confidence interval for the long-term average is 64%, as the lower bound, and 71%, as the higher bound, with an average value of about 68%. The DCC figure goes as high as 93% and as low as 15%, and there are several episodes of major correlation breakdowns in this window (i.e. from June 2016 to June 2020). The correlation on September 7, 2018 was around 72%. At the close-of-business (COB) of September 10, the DCC resulted in a sharp drop on the correlation (around 15%).<sup>9</sup> However, its figure didn't reflect fully the breakdown until a day later (i.e. September 11). After the Nasdaq event, there are three more major correlation breakdowns, one on August 1, 2019, and the other two on February 25, 2020, which is at the onset of the

<sup>9</sup> The DCC correlation at the COB September 10 is 53% based on the calibration using the time series up to September 10.

COVID period, and May 19, 2020, during the COVID period. The patterns of the breakdowns are all similar, i.e. have no warning signs prior to the sudden dislocation and convergences following it.

Figure 2: GARCH-DCC Dynamic Correlation from June 2016 to June 2020



## 5. Crisis replay and model comparison analysis

Li and Cheruvelil (2019) observe that risk metrics for balanced portfolios are quite sensitive to the correlation changes under the high correlation regime (e.g. above 75%).<sup>10</sup> As such, crisis replay represents an efficient way of illustrating how different statistical treatments of correlation can impact risk metrics. Five distinct models are calibrated using the time series up to the day before the crisis event or the crisis period. Following that, the models are calibrated every day for the selected window of the crisis, and used to calculate a theoretical margin for the next day. In addition to the Multivariate t-Copula, other models selected for comparison purposes are Historical Simulation VaR (HS VaR) and Filtered Historical Simulation VaR (FHS VaR).<sup>11</sup> For the FHS VaR models, decay factors ranging from 0.95 to 0.99 to “devol” and “revol”

<sup>10</sup> One approach to mitigate such effect could be the introduction of “correlation ceilings”.

<sup>11</sup> Gurrola-Perez and Murphy (2015) have in-depth discussions on these models, and have also specifically discussed the performance for these models on the spread positions (e.g. long Natural gas and Gasoil futures and short WTI).

the factor returns are considered.<sup>12</sup> The above models are also tested under the expected shortfall configuration (ES). All risk model consider a 99% quantile with 2-day liquidation period, and a portfolio composition of short 1 million MWh GAB and long 1 million MWh NRB on the 12-month expiry.

### 5.1. Nasdaq event

Table 4: Model Comparisons<sup>13</sup> and portfolio loss for 12M on September 10 and 11, 2018

Models or P/L	VaR/ES (\$ millions) (9/10/2018)	VaR/ES (\$ millions) (9/11/2018)	1-Day Shortfall (\$ millions)	2-Day Shortfall (\$ millions)
Portfolio P/L			(5.79)	(7.91)
HS VaR	2.62	3.21	(3.17)	(5.29)
FHS VaR ( $\lambda = 0.99$ )	2.12	2.54	(3.67)	(5.79)
FHS VaR ( $\lambda = 0.97$ )	2.13	2.63	(3.66)	(5.78)
Multivariate t-Copula VaR CCC	3.04	4.14	(2.75)	(4.87)
Multivariate t-Copula VaR DCC	3.03	5.27	(2.76)	(4.88)
HS ES	4.15	4.86	(1.64)	(3.76)
FHS ES ( $\lambda = 0.99$ )	3.84	4.65	(1.95)	(4.07)
FHS ES ( $\lambda = 0.97$ )	3.96	4.79	(1.83)	(3.95)
Multivariate t-Copula ES CCC	5.75	7.62	(0.04)	(2.16)
Multivariate t-Copula ES DCC	5.39	8.99	(0.40)	(2.52)

On September 7, 2018 the GAB contract price was \$54.27 per MWh, while the NRB contract was valued at \$44.59 per MWh. At the end of the first business day of the crisis, September 10, 2018, the 1-day loss of the theoretical portfolio is \$5.79 million. The cumulative 2-day loss amounts to \$7.91 million on September 11, 2018, the second day. Table 4 summarises the different risk metrics derived from the models discussed above, including the measurements of how much they fall short of the realized losses (i.e. shortfall). In particular, the table shows that the FHS VaR is the worst performer when the shortfall is considered. The unfiltered HS VaR performs better when compared with HS, but still with a sizable shortfall.<sup>14</sup> The multivariate t-Copula VaR with either CCC or DCC performs sizably better than either HS or FHS. This is not only related to the commonly known factors of fat tail distribution and robust tail interdependence with t-Copula, but also attributed to the GARCH's volatility forecast and for DCC only, correlation forecast.

The above results are in line with the observation in Gurrola-Perez and Murphy (2015) that the HS or FHS VaR models couldn't handle correlation breakdown very well, and that a correlation updating mechanism for these non-parametric models would be desirable. When the DCC and

<sup>12</sup> The "devol" and "revol" EWMA volatility start from the same date, i.e. June 6, 2016.

<sup>13</sup> Both GARCH-CCC and DCC in this table have GARCH volatility forecast. GARCH-DCC also uses correlation forecast.

<sup>14</sup> The reason that the unfiltered HS VaR performs better than the FHS VaR in this case is because there was a scenario of stress returns and correlation breakdown on August 1, 2018, just one month before the Nasdaq event.

CCC are compared, the former model doesn't generate higher margin than the CCC model. The main reason is that the Nasdaq event is a two-day stress event with no sign of correlation breakdown leading up to September 10, 2018. The DCC dynamic correlation (and the corresponding forecasted correlation) was actually higher (73%) than the CCC correlation (71%)<sup>15</sup> on September 7, 2018. The 99% ES for DCC and CCC are more robust than 99% VaR with HS and FHS models. This shows the DCC and CCC models are even more conservative at higher quantiles (e.g. 99.5%) than these non-parametric models.

## 5.2. COVID-19 Pandemic Event (March – May 2020)

Table 5: Model comparisons<sup>16</sup> and portfolio loss for 12M on May 15 and 18, 2020

Models or P/L	VaR/ES (\$ millions) (5/15/2020)	VaR/ES (\$ millions) (5/18/2020)	1-Day Shortfall (\$ millions)	2-Day Shortfall (\$ millions)
Portfolio P/L			(4.78)	(1.41)
HS VaR	2.17	2.38	(2.61)	0.76
FHS VaR ( $\lambda = 0.99$ )	2.22	1.88	(2.56)	0.81
FHS VaR ( $\lambda = 0.97$ )	2.46	2.08	(2.32)	1.05
Multivariate t-Copula VaR CCC	2.42	2.35	(2.36)	1.01
Multivariate t-Copula VaR DCC	2.46	3.22	(2.32)	1.05
HS ES	3.51	3.60	(1.27)	2.10
FHS ES ( $\lambda = 0.99$ )	3.66	3.51	(1.12)	2.25
FHS ES ( $\lambda = 0.97$ )	4.22	3.94	(0.56)	2.81
Multivariate t-Copula ES CCC	3.66	3.60	(1.12)	2.25
Multivariate t-Copula ES DCC	3.81	4.91	(0.97)	2.40

The COVID period was also marked by correlation breakdowns. Although it does not have the same extreme price scenario as seen during the Nasdaq event, on May 18, 2020 the correlation dropped to ~25%, gradually returning to its long-term value a week later. On this date the GAB contract recorded a 1% return, while NRB experienced -20% decrease. The 1-day portfolio loss was \$4.78MM, and two-day cumulative loss was much less (\$1.41MM) due to the NRB price bouncing back up on May 19, 2020.

Based on the prior discussion, the GARCH-DCC model has limited power in this case to forecast the new correlation regime. However, because the dynamic correlation could reflect the new correlation regime after the first day when the breakdown happens, GARCH-DCC model is still able to catch up to the lower correlation after the first day, which drives margin responsively higher for the rest of the low correlation regime. As such, in general, the multivariate t-copula

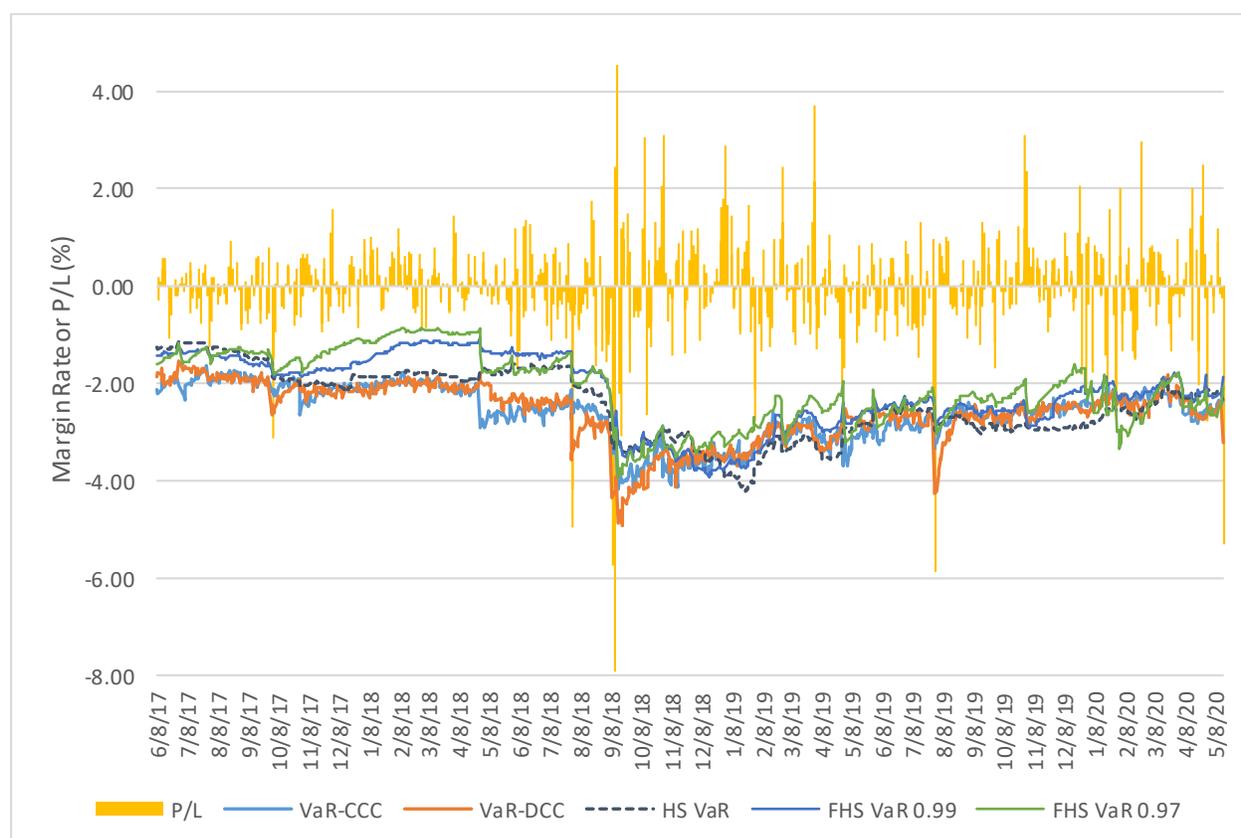
<sup>15</sup> The unconditional correlation based on the information up to September 7, 2018 is 71%. It drops to 68% after the COVID period in May and June 2020.

<sup>16</sup> Both GARCH-CCC and DCC in this table have GARCH volatility forecast. GARCH-DCC also uses correlation forecast

VaR or ES are more responsive to the correlation stress than the non-parametric models. See details in Table 5.

### 5.3. Backtesting analysis of various models

Figure 3: Model back-testing for the 5 selected models. Margin rate is calculated using 99% VaR compared with 2-day P/L. Period June 2017 to May 2020



In addition to the crisis replay, which focuses on specific days of the analysed period, another type of assessment that enables one to study the impact of correlation on risk metrics is back-testing. In such exercises, margins in the form of VaR 99% are calculated and compared with the 2-day P/L of the spread position. If in a particular day the margin is smaller than the P/L, then a breach or an exceedance is recorded. The widely adopted Kupiec likelihood ratio test is used to evaluate the frequency of exceedances statistically. The Kupiec test is defined by the null hypothesis that the expected proportion of violations is equal to 0.01.

The back-testing exercise performed for the same theoretical portfolio as the crisis replay, and considers the following three periods:

1. June 8, 2017 to September 7, 2018 – This is the period right before the Nasdaq event, and is categorized by a relatively calm market.

2. June 8, 2018 to May 18, 2020 – This is the period that includes both the Nasdaq event and the COVID-19 pandemic. It is categorized as a stress market period.
3. June 8, 2017 to May 18, 2020 – This is the whole period for the back-testing study.

From all three periods, the multivariate DCC and CCC perform more conservatively among these models, with DCC exhibiting the lowest number of breaches. The HS VaR has its Kupiec value close to the critical value. The two FHS VaR models fail the test. For the relative calm period, all three non-parametric models pass the Kupiec test with FHS VaR ( $\lambda = 0.99$ ) close to the critical threshold. However, for the stress period, HS VaR becomes really close to the borderline, while the two HFS VaR models fail the test. All test results are displayed in Tables A7 to A9 in the Appendix, with a visual illustration in Figure 3.

## 6. Conclusion and further work

The paper studied the correlation behaviour during adverse market conditions using the GARCH-DCC model to filter out the non-stationary statistical properties of price returns. In particular, a case study based on the German and Nordic electricity power futures, and the spread positions comprising of them, was considered. These electricity power futures exhibit certain salient statistical characteristics such as fat tail, volatility clustering, low autocorrelation, low skewness and sudden correlation breakdown (and convergence) among them. The study focuses on the dynamic correlations and correlation breakdowns from these spread positions. The use of a fat tail and volatility clustering via student-t copula, i.e. GARCH volatility framework and dynamic DCC correlations, allow us to single out the correlation behaviour. The shocks observed during the Nasdaq event in September 2018 and COVID-19 pandemic in early 2020 are the focus of the analysis.

Amongst other aspects discussed previously, the paper concludes that correlation breakdowns seem to be more frequent in the electricity power market than expected. Even though the nature of the shocks differs between the Nasdaq event and the COVID-19 period, the pattern of the correlation breakdown was relatively similar. It is also important to highlight that the correlation between GAB and NRB varies through time, exhibiting different structures for distinct expiry dates. In order to illustrate the importance of the modelling techniques to capture correlation, a crisis replay and back-testing exercises are performed. The multivariate t-copula DCC or CCC with volatility and correlation forecasts demonstrate more conservative metrics in terms of coverage rate as well as breaching amount during the Nasdaq event and COVID-19 period when compared with several non-parametric models. Although not part of the study, both DCC and CC models provide a naturally conservative and anti-procyclical floor during a relative calm period.

Several lines of further work are worth to explore to continue to improve model performance for spread positions from different asset classes in a practical manner. Typical spread positions such as Treasury cash and futures position, SPX option and VIX futures position are going to be included in the further study. Certain unique features to an asset class (e.g. seasonal effect in certain futures products) shall be added to the framework. From the standpoint of procyclicality,

since the GARCH-DCC copula framework would inevitably increase the margin responsiveness due to correlation breakdown, an effective margin floor may need to be explored to make margin less procyclical. A volatility floor (e.g. long term volatility) is commonly used for this purpose. For spread positions with dynamic correlation assumption, a concept of correlation ceiling<sup>17</sup> can be explored to give spread positions a reasonable level of offset, and yet maintain sufficient level of margin.

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<sup>17</sup> For the spread position studied in this paper, the correlation is in the range of [0.15, 0.93]. If assuming correlation level follows a gamma distribution, very intuitively, certain high quantile (e.g. 95%) could be used as a correlation ceiling.

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## 8. Appendix

### 8.1. Statistical properties of time series

Table A1: Statistics – Prices between June 2016 to June 2020.

NAME	nobs	mean	min	max	std	skewness	kurtosis
GAB12M	1039	39.20	20.79	58.03	8.99454	-0.00658	-0.82701
NRB12M	1039	30.39	25.22	48.69	7.24175	0.50041	-0.05365

Table A2: Statistics – Returns between June 2016 to June 2020

NAME	nobs	mean	min	max	std	skewness	kurtosis
GAB12M	1039	0.00030	-0.14859	0.08975	0.01995	-1.06303	9.82460
NRB12M	1039	-0.00002	-0.21084	0.17371	0.02653	-1.31483	15.66271

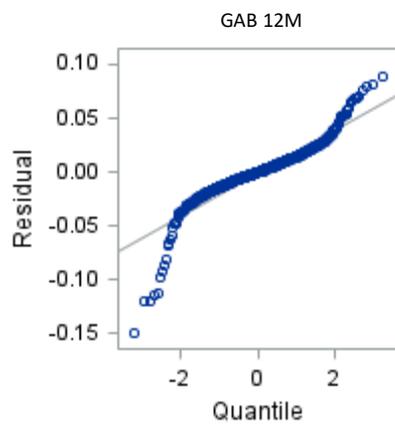


Figure A1: Q-Q plot for GAB 12M residual

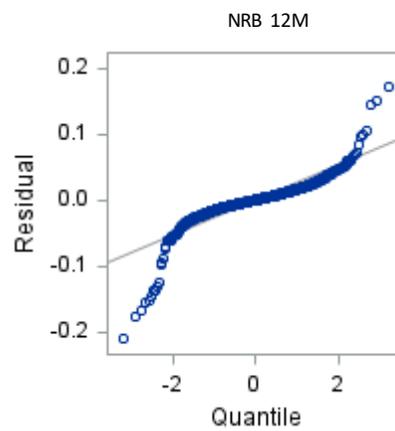


Figure A2: Q-Q plot for NRB 12M residual

## 8.2. Model parameters

Table A3: GARCH type models with different types of innovations – GAB12m

Volatility Model	Innovation	$\omega$	$\alpha$	$\gamma (GJR)$	$\beta$	$AR1$	$\beta_s (Skew)$	$\nu (DoF)$	LL
<b>GARCH(1,1)</b>	<b>Student-t</b>	<b>0.0000144</b>	<b>0.0113</b>		<b>0.9869</b>	<b>-0.020</b>		<b>2.11</b>	<b>1588</b>
		<b>(0.0000253)</b>	<b>(0.00612)</b>		<b>(0.00639)</b>			<b>(0.03)</b>	
GJR- GARCH(1,1)	Student-t	0.00007578	0.06636	-0.06636	0.7332	-0.020		3.42	1560
		(0.0000244)	(0.043300)	(0.05200)	(0.09591)			(0.53)	
GARCH(1,1)	Skewed t	0.0004640	0.32258		0.2073	-0.020	0.02244	2.54	1567
		(0.0001528)	(0.207310)		(0.07928)		(0.04381)	(0.24)	
GJR- GARCH(1,1)	Skewed t	0.0000612	0.055433	-0.05543	0.7810	-0.020	0.028639	3.44	1560
		(0.0000168)	(0.034216)	(0.041540)	(0.06506)		(0.047657)	(0.51)	

Note: all model parameters are calibrated using time series up to September 7, 2018

Table A4: GARCH type models with different types of innovations – NRB12m

Volatility Model	Innovation	$\omega$	$\alpha$	$\gamma (GJR)$	$\beta$	$AR1$	$\beta_s (Skew)$	$\nu (DoF)$	LL
<b>GARCH(1,1)</b>	<b>Student-t</b>	<b>6.8664E-6</b>	<b>0.0141</b>		<b>0.9858</b>	<b>0.026</b>		<b>2.67</b>	<b>1515</b>
		<b>(2.1313E-6)</b>	<b>(0.0047)</b>		<b>(0.0057)</b>			<b>(0.35)</b>	
GJR- GARCH(1,1)	Student-t	0.00001	0.0009	0.02953	0.9746	0.026		2.50	1511
		(1.9192E-6)	(0.0130)	(0.02652)	(0.0081)			(0.08)	
GARCH(1,1)	Skewed t	0.0000058	0.0164		0.9796	0.026	-0.02402	2.47	1512
		(3.5834E-6)	(0.05114)		(0.0648)		(0.04207)	(1.11)	
GJR- GARCH(1,1)	Skewed t	0.0000671	0.00001	0.03778	0.8859	0.026	-0.01600	2.55	1508
		(0.0000226)	(0.21675)	(0.20300)	(0.0517)		(0.03974)	(0.78)	

Note: all model parameters are calibrated using time series up to September 7, 2018

Table A5: GARCH(1,1) with t innovation – GAB12m Post March 2020

Volatility Model	Innovation	$\omega$	$\alpha$	$\gamma (GJR)$	$\beta$	$AR1$	$\beta_s (Skew)$	$\nu (DoF)$	LL
<b>GARCH(1,1)</b>	<b>Student-t</b>	<b>7.65E-6</b>	<b>0.0301</b>		<b>0.9625</b>	<b>0.003</b>		<b>2.74</b>	<b>2748</b>
		<b>(5.53E-6)</b>	<b>(0.0127)</b>		<b>(0.0155)</b>			<b>(0.14)</b>	

Note: all model parameters are calibrated using time series up to June 1, 2020

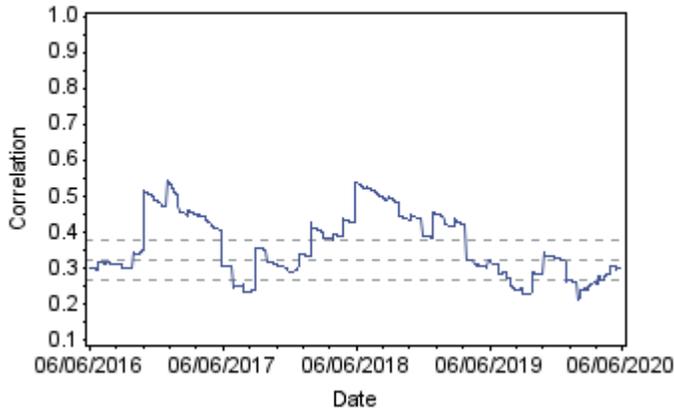
Table A6: GARCH(1,1) with t innovation – NRB12m Post March 2020

Volatility Model	Innovation	$\omega$	$\alpha$	$\gamma$ (GJR)	$\beta$	AR1	$\beta_s$ (Skew)	$\nu$ (DoF)	LL
<b>GARCH(1,1)</b>	<b>Student-t</b>	<b>1.93E-5</b> <b>(1.99E-5)</b>	<b>0.0198</b> <b>(0.0116)</b>		<b>0.9726</b> <b>(0.0153)</b>	<b>0.001</b>		<b>2.30</b> <b>(0.21)</b>	<b>2577</b>

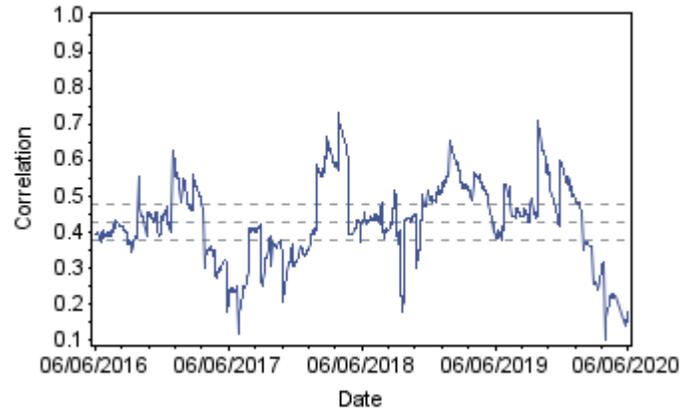
Note: all model parameters are calibrated using time series up to June 1, 2020

### 8.3. Correlation breakdowns

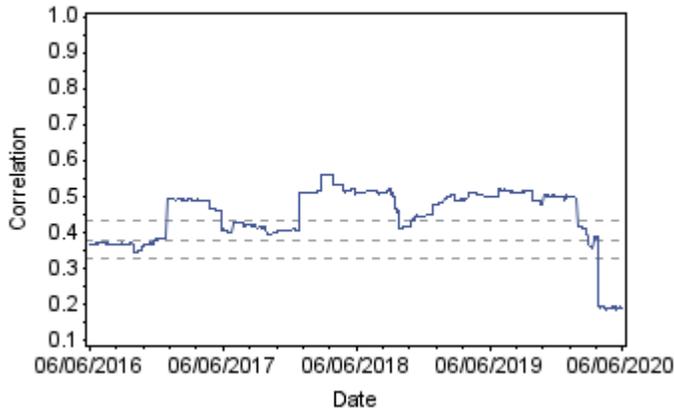
**DCC Correlation: GAB1m vs NRB1m**



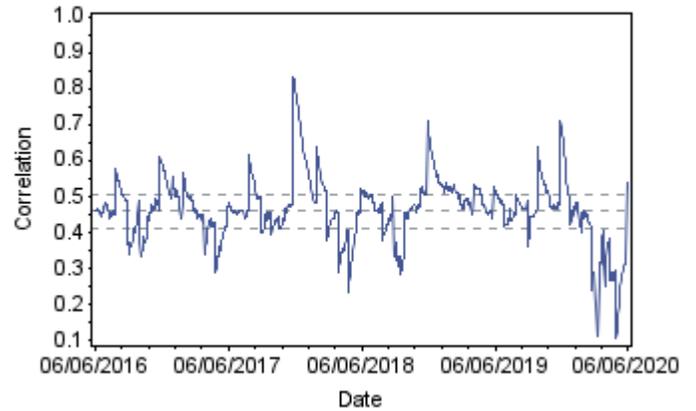
**DCC Correlation: GAB2m vs NRB2m**



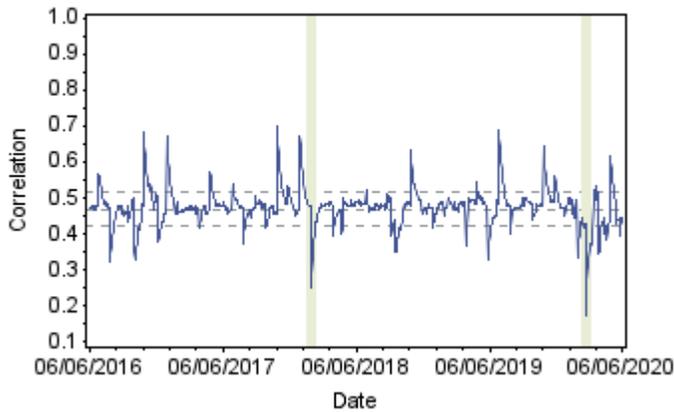
**DCC Correlation: GAB3m vs NRB3m**



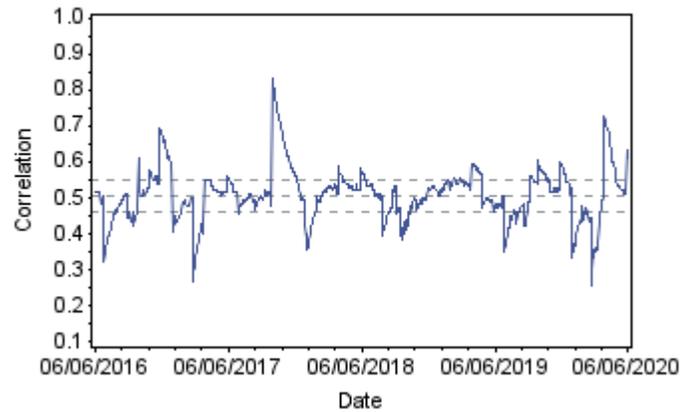
**DCC Correlation: GAB4m vs NRB4m**



**DCC Correlation: GAB5m vs NRB5m**



**DCC Correlation: GAB6m vs NRB6m**



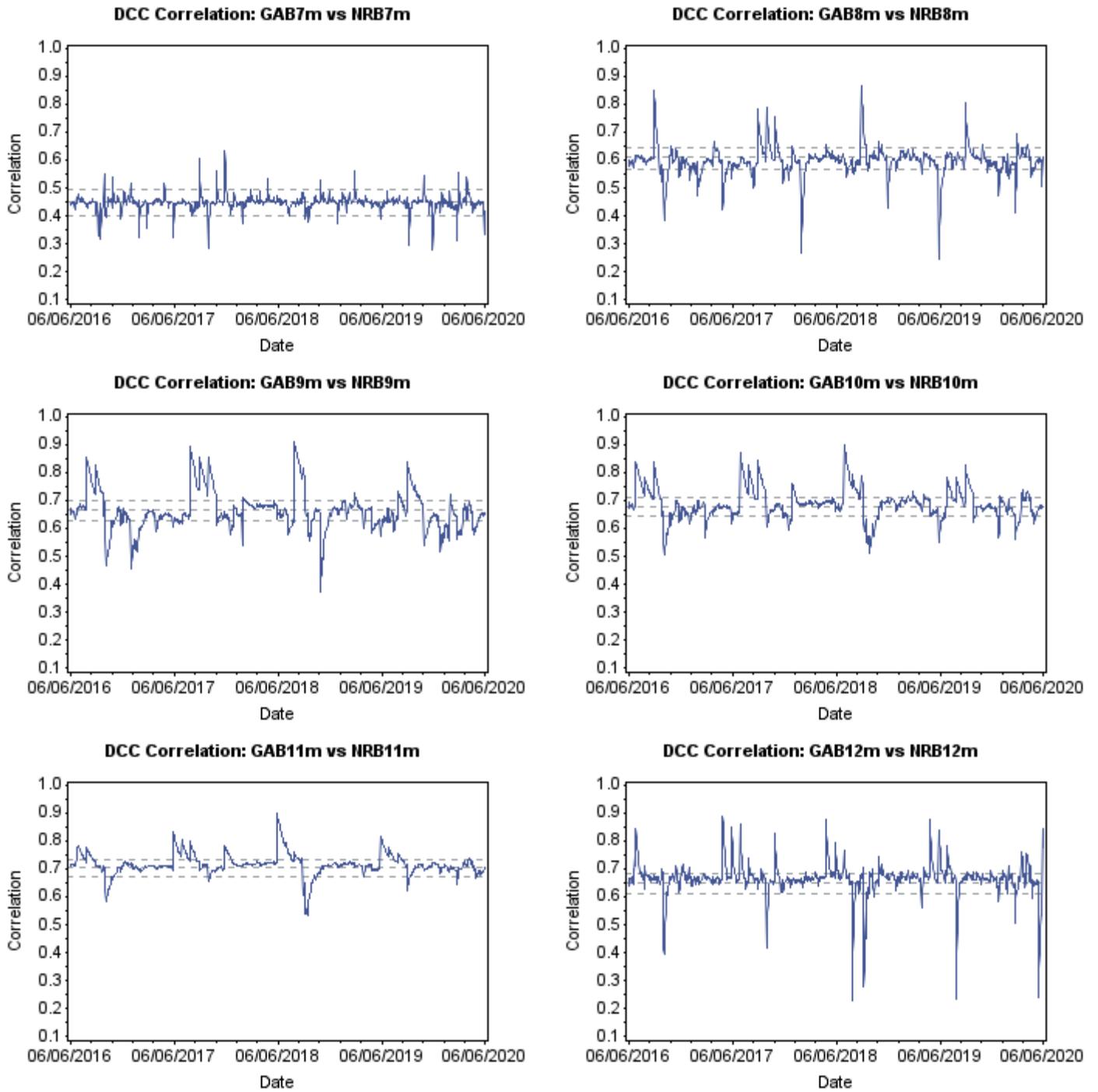


Figure A3: DCC Correlation for 1M to 12M Expiries.

## 8.4. Other Backtesting results

Table A7: VaR backtesting for long Nordic and short German position during pre Nasdaq and COVID period (June 8, 2017–September 7, 2018)

Models	Exceedance	Kupiec Statistics	p-Value
Multivariate t-copula DCC	5	0.8283	0.3628
Multivariate t-copula CCC	5	0.8283	0.3628
HS VaR	7	3.3090	0.0689
FHS VaR ( $\lambda = 0.97$ )	7	3.3090	0.0689
FHS VaR ( $\lambda = 0.99$ )	8	5.0129	0.0252

Table A8: VaR backtesting for long Nordic and short German position during Nasdaq and COVID period (June 8, 2018–May 18, 2020)

Models	Exceedance	Kupiec Statistics	p-Value
Multivariate t-copula DCC	9	2.6452	0.1039
Multivariate t-copula CCC	10	3.9544	0.0467
HS VaR	11	5.4680	0.0194
FHS VaR ( $\lambda = 0.97$ )	<b>14</b>	<b>11.0676</b>	<b>0.0009</b>
FHS VaR ( $\lambda = 0.99$ )	<b>15</b>	<b>13.2436</b>	<b>0.0003</b>

Table A9: VaR backtesting for long Nordic and short German position during pre and post Nasdaq and COVID period (June 8, 2017–May 18, 2020)

Models	Exceedance	Kupiec Statistics	p-Value
Multivariate t-copula DCC	10	0.7222	0.3954
Multivariate t-copula CCC	11	1.3864	0.2390
HS VaR	14	4.4288	0.0353
FHS VaR ( $\lambda = 0.97$ )	17	<b>8.7912</b>	<b>0.0030</b>
FHS VaR ( $\lambda = 0.99$ )	17	<b>8.7912</b>	<b>0.0030</b>